

AN INVOLUTION ACTING NONTRIVIALY ON HEEGARD-FLOER HOMOLOGY

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ABSTRACT. We show that a certain involution on a homology sphere Σ induces a nontrivial homomorphism on its Heegard-Floer homology groups (recently defined by Ozsváth and Szabó). We discuss application of this to constructing exotic smooth structures on 4-manifolds.

0. INTRODUCTION

Let W be the smooth contractible 4-manifold consisting of a single 1- and 2- handle attached as in Figure 1 (one of the Mazur manifolds).

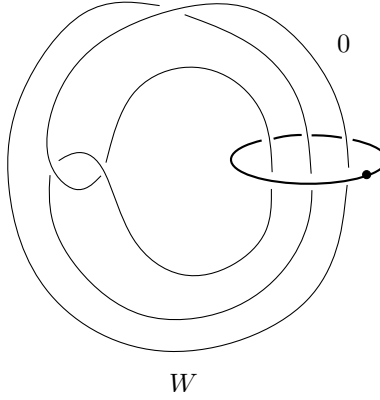


FIGURE 1.

The boundary is a homology sphere $\Sigma = \partial W$. Let $f : \Sigma \rightarrow \Sigma$ be the obvious involution obtained by first surgering $S^1 \times B^3$ to $B^2 \times S^2$ in the interior of W , then surgering the other imbedded $B^2 \times S^2$ back to $S^1 \times B^3$ (i.e. replacing the dots in Figure 1). This involution interchanges the small linking loops of the two circles in Figure 1. It is known that this involution acts nontrivially on the Donaldson-Floer

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homology of Σ , (c.f. [A1], [S]). In this paper we will show that the involution f also acts nontrivially on the Heegard-Floer homology group $HF^+(\Sigma)$ of Σ , which was defined in [OS1]. We will deduce the proof from the construction of [A2] and the 4-manifold invariants of [OS3]. This gives a possible way of changing smooth structures of 4-manifolds that contain W (such codimension zero contractible submanifolds are usually called “corks”, e.g. [K], [M], [S], and they can be made Stein [AM]).

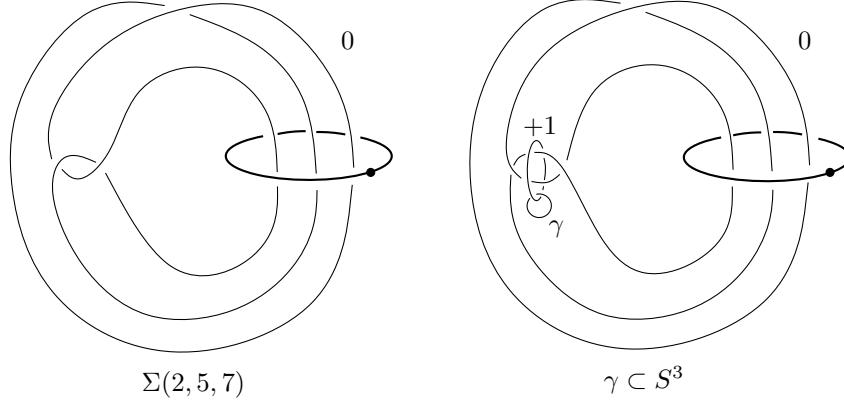


FIGURE 2.

By [AK], the boundary of the 4-manifold in the first picture of the Figure 2 is the Brieskorn homology sphere $\Sigma(2, 5, 7)$, which we can relate to Σ as follows: The second picture of Figure 2 is obtained by attaching a +1 framed 2-handle to W , hence the boundary is S^3 (since the 0-framed 2-handle slides over the +1 framed handle, and cancels the 1-handle). So, in this picture, γ is just a loop in S^3 , which is drawn nonstandardly. It is easily seen that in the standard picture of S^3 , the loop γ corresponds to the $(-3, 3, -3)$ pretzel knot $K \subset S^3$ as shown in Figure 3 (e.g. [AK]).

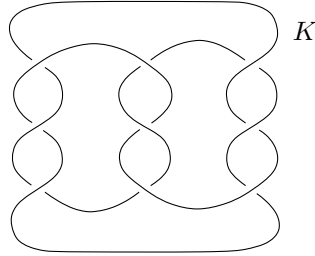


FIGURE 3.

Note that doing $r+1$ surgery to γ corresponds to doing r surgery to K in S^3 , which we will denote by $S_r^3(K)$. Hence $S_1^3(K) = \Sigma(2, 5, 7)$ and $S_{-1}^3(K) = \Sigma$. We will use this identifications in the Heegaard-Floer homology calculations of the next section.

1. CALCULATING THE HEEGAARD-FLOER HOMOLOGY $HF^+(\Sigma)$

We first calculate the Heegaard-Floer homology of $\Sigma(2, 5, 7)$ using techniques of [OS5], from this and the surgery exact sequence of [OS2] we will deduce the Heegaard-Floer homology of Σ .

From [AK] we know that $\Sigma(2, 5, 7)$ is the boundary of the negative definite plumbing of disk bundles over 2-spheres described by the following plumbing graph:

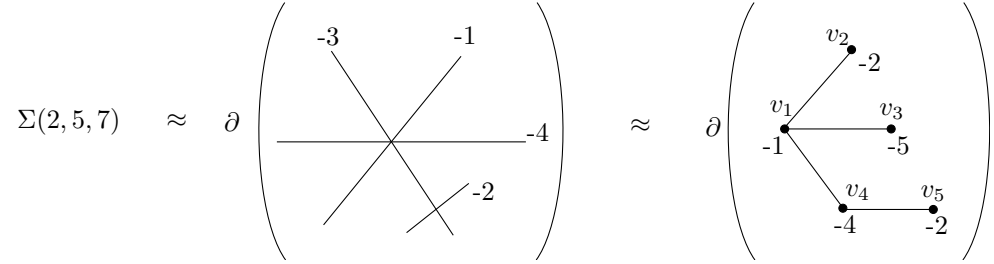


FIGURE 4.

This graph is negative definite and has only one bad vertex v_1 in the sense of [OS5], i.e., only $[v_1] \cdot [v_1] > -d(v_1)$ where $d(v)$ counts edges containing v .

In the notation of [OS5] we write $\Sigma(2, 5, 7) = \partial X(G) = Y(G)$, and we compute

$$HF^+(-Y(G)) \cong H^+(G)$$

where $H^+(G)$ is a group which can be calculated by the algorithm of [OS5] for negative definite (possibly disconnected) trees G with at most one bad vertex. Here we follow this algorithm (for the terminology, we refer to [OS5]):

First we need to find characteristic vectors K satisfying

$$(1) \quad [v] \cdot [v] + 2 \leq \langle K, [v] \rangle \leq -[v] \cdot [v].$$

We remark that there are only finitely many such vectors since G is a definite graph, in particular there are 80 characteristic vectors satisfying (1) for the given graph. Proposition 3.2 in [OS5] characterizes a spanning set for $K^+(G)$ which is the dual point of view for $H^+(G)$. Among the 80 characteristic vectors satisfying (1) we look for those vectors K which carry full paths ending at vectors L for which $-L$ satisfies (1). There are precisely three such vectors K and we list them and also the full paths for convenience (paths are obtained by consecutively adding $2PD(v_i)$ for the numbers i listed):

$$\begin{aligned}
K_1 &= 12v_1 + 6v_2 + 3v_3 + 4v_4 + 2v_5 & \text{path: } 1,2,1 \\
K_2 &= -8v_1 - 4v_2 - v_3 - 2v_4 - 2v_5 & \text{path: } 1,2,1,5,4,1,2,1,3,1,2,1,4,1,2,1,5 \\
K_3 &= -16v_1 - 8v_2 - 3v_3 - 4v_4 - 2v_5 & \text{path: } 1,2,1
\end{aligned}$$

All other equivalence classes in $K^+(G)$ can be obtained using the U -action from these vectors. We must check which are distinct. From now on, we will write the vectors as 5-tuples with entries $\langle K, [v_i] \rangle$, hence K_i will be denoted as

$$(1, 0, -3, -2, 0), (1, 0, -3, -2, 2), \text{ and } (1, 0, -1, -2, 0).$$

The following diagram shows how $U \otimes K_1$, $U \otimes K_2$ and $U \otimes K_3$ are equivalent to $(-3, 4, 1, 2, 0)$, and hence to each other:

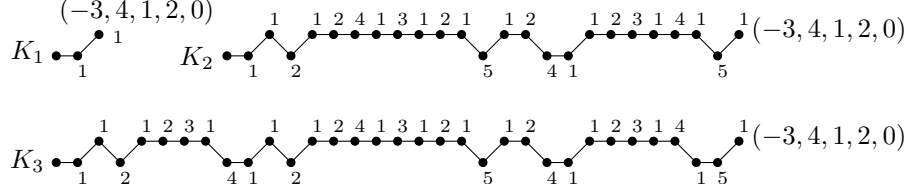


FIGURE 5.

In the above figure, vertices correspond to characteristic vectors and an edge ending with a number i means $2PD(v_i)$ was added to previous vector to reach the new one. Edges with positive slope mean tensoring with U . Hence first diagram represents the vectors $(1, 0, -3, -2, 0), (-1, 2, -1, 0, 0), (-3, 4, 1, 2, 0)$ which tells us that

$$(1, 0, -3, -2, 0) \sim (-1, 2, -1, 0, 0) \text{ and } U \otimes (-1, 2, -1, 0, 0) \sim (-3, 4, 1, 2, 0)$$

Hence each K_i lies on the same grading level and we can check this by calculating

$$\frac{K \cdot K + |G|}{4}$$

which gives zero for each K_i . Therefore we conclude: $HF^+(-Y(G)) \cong T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)}$ By Prop 7.11 of [OS3] we can convert this to the Heegard-Floer homology of $Y(G)$:

$$HF_*^+(Y(G)) = HF_-^{*-2}(-Y(G)) = T_0^+ \oplus \mathbb{Z}_{(-1)} \oplus \mathbb{Z}_{(-1)}$$

To do this, we first use the following long exact sequence to compute $HF^-(-Y(G))$:

$$\dots \rightarrow HF^-(-Y(G)) \longrightarrow HF^\infty(-Y(G)) \longrightarrow HF^+(-Y(G)) \rightarrow \dots$$

then we adjust gradings by changing the signs and subtracting -2 . Note that here we adapted the convention of [OS5] by denoting $T_s^+ = \bigoplus_{k=0}^{\infty} \mathbb{Z}_{(s+2k)}$. So we have:

$$HF^+(\Sigma(2, 5, 7)) = T_0^+ \oplus \mathbb{Z}_{(-1)} \oplus \mathbb{Z}_{(-1)}$$

Lemma 1. $HF^+(\Sigma) \cong T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)}$.

Proof. Let us consider the Heegard-Floer homology long exact sequence of [OS2] for the $(-3, 3, -3)$ pretzel knot $K \subset S^3$:

$$\dots \rightarrow HF^+(S^3) \longrightarrow HF^+(S_0^3(K)) \longrightarrow HF^+(S_1^3(K)) \rightarrow \dots$$

Recall that the middle two arrows in the sequence decrease the gradings by $1/2$. Since $S_1^3(K) = \Sigma(2, 5, 7)$ we know 2 out of 3 terms of the exact sequence, from this we can compute the third term $HF^+(S_0^3(K)) = T_{(1/2)}^+ \oplus T_{(-1/2)}^+ \oplus \mathbb{Z}_{(-1/2)} \oplus \mathbb{Z}_{(-1/2)}$, as shown in the diagram:

$$\begin{array}{ccccc}
 \dots \rightarrow HF^+(S^3) & \longrightarrow & HF^+(S_0^3(K)) & \longrightarrow & HF^+(\Sigma(2, 5, 7)) \rightarrow \dots \\
 \vdots & & \vdots & & \vdots \\
 \mathbb{Z}_{(4)} & \searrow & \mathbb{Z}_{(9/2)} & \searrow & \mathbb{Z}_{(4)} \\
 \mathbb{Z}_{(2)} & \searrow & \mathbb{Z}_{(7/2)} & \searrow & \mathbb{Z}_{(2)} \\
 \mathbb{Z}_{(0)} & \xrightarrow{g} & \mathbb{Z}_{(3/2)} & \searrow & \mathbb{Z}_{(0)} \\
 & \text{nonzero} & \mathbb{Z}_{(1/2)} & \searrow & \mathbb{Z}_{(-1)} \oplus \mathbb{Z}_{(-1)} \\
 & & \mathbb{Z}_{(-1/2)} \oplus \mathbb{Z}_{(-1/2)} \oplus \mathbb{Z}_{(-1/2)} & \searrow &
 \end{array}$$

Now by plugging in this value of $HF^+(S_0^3(K))$ in the Heegard-Floer homology exact sequence of the knot K in S^3 below (this time involving -1 , 0 , and no surgeries), we calculate the Heegard -Floer homology of $\Sigma = S_{-1}^3(K)$ to be

$$\begin{array}{ccccc}
 HF^+(\Sigma) \cong T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} & & & & \\
 \dots \rightarrow HF^+(S_{-1}^3(K)) & \longrightarrow & HF^+(S_0^3(K)) & \longrightarrow & HF^+(S^3) \rightarrow \dots \\
 \vdots & & \vdots & & \vdots \\
 \mathbb{Z}_{(4)} & \searrow & \mathbb{Z}_{(9/2)} & \searrow & \mathbb{Z}_{(4)} \\
 \mathbb{Z}_{(2)} & \searrow & \mathbb{Z}_{(7/2)} & \searrow & \mathbb{Z}_{(2)} \\
 \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} & \searrow & \mathbb{Z}_{(3/2)} & \searrow & \mathbb{Z}_{(0)} \\
 & & \mathbb{Z}_{(1/2)} & \searrow & \xrightarrow{h} \text{zero} \\
 & & \mathbb{Z}_{(-1/2)} \oplus \mathbb{Z}_{(-1/2)} \oplus \mathbb{Z}_{(-1/2)} & \searrow &
 \end{array}$$

In these calculations we used the fact that all the groups are equipped with U -action (an action of $\mathbb{Z}[U]$ which lowers the grading by -2), and the maps are equivariant with respect to this action.

Also we used two additional facts, which were explained to us by P. Ozsváth: The lower left map g of the first diagram is nonzero, and the lower right map h of the second diagram is zero. Explanation: By using Theorem 1.4 of [OS6] and Theorem 6.1 of [OS7] we can calculate $HF^+(S_0^3(K))$ independently, and by the exact sequence and the fact that g has to lower the degree by $1/2$ implies that g must be nonzero. The map h is zero because it is obtained by summing over the maps induced by a cobordism (Z, s) with $Spin^c$ structures s (extending the one on the boundary). Z has signature -1 and there are two such $Spin^c$ structures which cancel each other. \square

2. HEEGARD-FLOER HOMOLOGY ACTION ON W

Let W_0 be the punctured W (i.e. $W - B^4$). We want to compute the map induced by the cobordism W_0 [OS2]

$$F_{W_0}^+ : HF^+(\Sigma) \rightarrow HF^+(S^3).$$

W consists of $S^1 \times B^3$ with a 2-handle attached. By turning W upside down we see W_0 is a union of cobordisms W_1 from Σ to $S^1 \times S^2$, and W_2 from $S^1 \times S^2$ to S^3 (a 3-handle). Hence we have the decomposition $F_{W_0}^+ = F_{W_2}^+ \circ F_{W_1}^+$. The map $F_{W_2}^+ : HF^+(S^1 \times S^2) \rightarrow HF^+(S^3)$ is well understood, it is obtained by the projection

$$T_{1/2}^+ \oplus T_{-1/2}^+ \rightarrow T_{-1/2}^+ \xrightarrow{\cong} T_0^+$$

The second map is U -equivariant, in particular it maps the lowest degree generator $Z_{(-1/2)}$ of $T_{-1/2}^+$ to the lowest degree generator $Z_{(0)}$ of T_0^+ . So it is sufficient to understand the map $F_{W_1}^+ : HF^+(\Sigma) \rightarrow HF^+(S^1 \times S^2)$. Notice W_1 is obtained by attaching a 2-handle to $\Sigma = \partial W$ along the loop δ with 0-framing (see Figure 6).

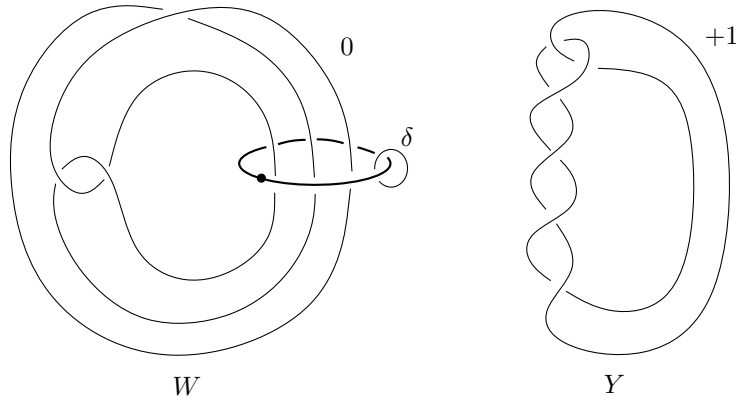


FIGURE 6.

Denote the manifold obtained from Σ , doing k -surgery to δ , by $\Sigma_k(\delta)$. Note that $\Sigma_0(\delta) = S^1 \times S^2$ and $\Sigma_1(\delta)$ is the second manifold of Figure 6, which we will call Y to simplify the notation. We can write the corresponding Heegard-Floer homology exact sequence of $\delta \subset \Sigma$:

$$(2) \quad \dots \rightarrow HF^+(\Sigma) \longrightarrow HF^+(\Sigma_0(\delta)) \longrightarrow HF^+(Y) \rightarrow \dots$$

$F_{W_1}^+$ is just the first map in the exact sequence, to compute it we need to know the third term in the sequence. To calculate $HF^+(Y)$ we use the Heegard-Floer exact sequence of the knot $\beta \subset Y$ of Figure 7.

$$\dots \rightarrow HF^+(Y) \longrightarrow HF^+(Y_0(\beta)) \longrightarrow HF^+(Y_1(\beta)) \rightarrow \dots$$

It is easy to check that $Y_0(\beta)$ is the manifold obtained by 0-surgery to the figure eight knot, and $Y_1(\beta)$ is $-\Sigma(2, 3, 7)$. In [OS4], [OS5] homologies of these were computed : $HF^+(Y_0(\beta)) = T_{1/2}^+ \oplus T_{-1/2}^+ \oplus \mathbb{Z}_{(-1/2)}$ and $HF^+(-\Sigma(2, 3, 7)) = T_0^+ \oplus \mathbb{Z}_{(0)}$. By plugging these to the last exact sequence we compute

$$HF^+(Y) = T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)}$$

By plugging the value $HF^+(Y)$ in the previous exact sequence (2) we see that the

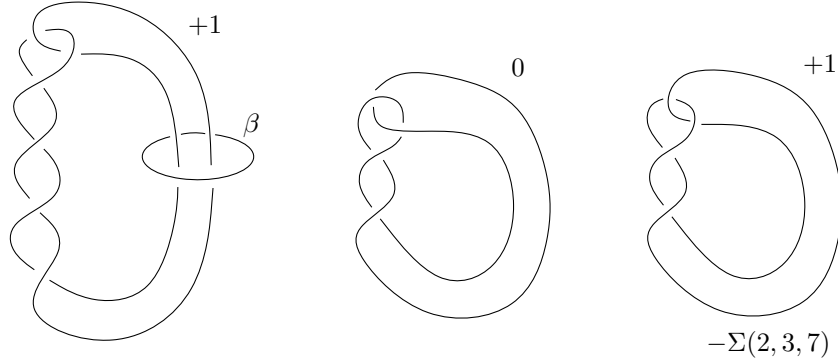


FIGURE 7.

map $F_{W_1}^+ : HF^+(\Sigma) \rightarrow HF^+(S^1 \times S^2)$ is given by the obvious projection and the inclusion (where the middle map is an U -equivariant isomorphism)

$$T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \rightarrow T_0^+ \xrightarrow{\cong} T_{-1/2}^+ \rightarrow T_{-1/2}^+ \oplus T_{1/2}^+$$

Hence the map $F_{W_0}^+ : HF^+(\Sigma) \rightarrow HF^+(S^3)$ is given by the obvious projection

$$(3) \quad T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \rightarrow T_0^+$$

3. NONTRIVIALITY OF THE INVOLUTION f

Let Z be the $K3$ surface constructed in [A1]. It was shown that $Z \# \overline{\mathbb{C}P^2}$ decomposes as the union of two codimension zero submanifolds glued along their common boundaries

$$Z \# \overline{\mathbb{C}P^2} = N \cup W$$

where W is the Mazur manifold of Figure 1, furthermore by identifying N and W via the involution $f : \Sigma \rightarrow \Sigma$ (instead of the identity) gives the decomposed manifold

$$N \cup_f W = 3(\mathbb{C}P^2) \# 20\overline{\mathbb{C}P^2}$$

Now let us remember the 4-manifold invariants of [OS3]: Let (X, s) be a smooth closed 4-manifold with a $Spin^c$ structure s such that it is decomposed as a union of two codimension zero submanifolds glued along their boundaries $X = X_1 \cup_{\partial} X_2$ with $b_2^+(X_i) > 0$, $i = 1, 2$. Call $Y = \partial X_i$. By puncturing each X_i in the interior we get two cobordisms: X'_1 from S^3 to Y , and X'_2 from Y to S^3 .

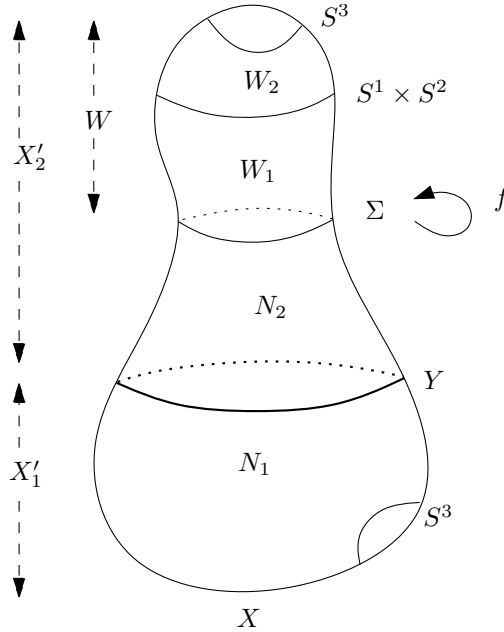


FIGURE 8.

Since $b_2^+(X'_i) > 0$ maps $F_{X'_1}^\infty$ and $F_{X'_2}^\infty$ induced by cobordism are zero (upper right and lower left vertical arrows in the diagram), hence the following commuting diagram

$$\begin{array}{ccccccc}
& & \longrightarrow & HF^+(S^3) & \xrightarrow{\delta} & HF^-(S^3) & \longrightarrow HF^\infty(S^3) \longrightarrow \dots \\
& & & \downarrow & & \Downarrow F_{X'_1}^- & \downarrow \text{zero} \\
\dots \longrightarrow & HF^\infty(Y) & \longrightarrow & HF^+(Y) & \implies & HF^-(Y) & \longrightarrow HF^\infty(Y) \longrightarrow \dots \\
& \downarrow \text{zero} & & \Downarrow F_{X'_2}^+ & & \downarrow & \downarrow \\
\dots \longrightarrow & HF^\infty(S^3) & \longrightarrow & HF^+(S^3) & \longrightarrow & HF^-(S^3) & \longrightarrow HF^\infty(S^3) \longrightarrow \dots
\end{array}$$

induces a well defined map: $F_{(X,s)}^{mix} : HF^-(S^3) \rightarrow HF^+(S^3)$. Then the Ozsváth-Szabó invariant $\Phi_{(X,s)} \in \mathbb{Z}$ is the degree of this map on $\mathbb{Z}_{(-2)}$ generator (in particular the invariant doesn't depend on the splitting of X). By [OS4], [OS8] we know that this invariant is zero on $3(\mathbb{CP}^2) \# 20\overline{\mathbb{CP}^2}$ while it is non-zero on $K3 \# \overline{\mathbb{CP}^2}$ (actually to get this conclusion we also need to show that either the homotopy $K3$ of [A2] is diffeomorphic to actual $K3$, or directly apply the technique of [OS8] to show that it has nonzero invariant - either way works.). Also, since this invariant is independent of splittings, we can further split $X = Z \# \overline{\mathbb{CP}^2}$ as shown in Figure 8, and decompose

$$F_{X'_2}^+ = F_{W_0}^+ \circ F_{N_2}^+$$

We can view the manifold $N \cup_f W$ as being obtained from $N \cup W$ by cutting it along Σ and sticking in the cobordism manifold $H = \Sigma \times [0, 1/2] \cup_f \Sigma \times [1/2, 1]$. Also the involution f induces a map $f^* : HF^+(\Sigma) \rightarrow HF^+(\Sigma)$ via the cobordism H , ($f^* = F_H^+$ in [OS3]'s notation). So the nonzero invariant of $Z \# \overline{\mathbb{CP}^2}$ is obtained by computing the degree of the map:

$$\mathbb{Z}_{(-2)} \xrightarrow{\lambda} T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \rightarrow T_0^+$$

(where λ is some map and the second map is the projection) whereas the zero invariant of $3(\mathbb{CP}^2) \# 20\overline{\mathbb{CP}^2}$ is obtained by computing the degree of the map:

$$\mathbb{Z}_{(-2)} \xrightarrow{\lambda} T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \xrightarrow{f^*} T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)} \rightarrow T_0^+$$

Note that since H is diffeomorphic to the product $\Sigma \times [0, 1]$ the U -equivariant map f^* should look like the identity map. At first glance this appears to contradict the above result. The explanation is that, the two ends of H are different copies of Σ , only when we fix the both ends by a reference copy of Σ , then f^* becomes an isomorphism permuting two of the zero degree generators of $T_0^+ \oplus \mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)}$. More specifically, f^* permutes the zero degree generator of T_0^+ with one of the generators of the remaining $\mathbb{Z}_{(0)} \oplus \mathbb{Z}_{(0)}$. So, actually f^* is only a U -equivariant isomorphism, which happens to differ from the identity isomorphism. Hence we proved:

Theorem 2. $f : \Sigma \rightarrow \Sigma$ induces a nontrivial involution $f^* : HF^+(\Sigma) \rightarrow HF^+(\Sigma)$.

If (M, s) is any smooth cobordism from S^3 to Σ with $Spin^c$ -structure s , and $b_2^+(M) > 1$, then Ozsváth-Szabó procedure gives a map $F_{(M,s)}^{mix} : HF^-(S^3) \rightarrow HF^+(\Sigma)$. Previous theorem gives a potential way of detecting exotic smoothings of 4-manifolds.

Corollary 3. *Let $Q = N \cup_{\partial} W$ be a smooth closed 4-manifold with $b_2^+ > 1$ (the union is taken along the common boundary Σ). Let N_0 be the cobordism from S^3 to Σ obtained from N by removing a copy of B^4 from its interior. Then the manifold $Q' = N \cup_f W$ is a fake copy of Q , provided that the image of the map $F_{(N_0,s)}^{mix} : HF^-(S^3) \rightarrow HF^+(\Sigma)$ lies in T_0^+ for some s .*

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